Non-reciprocal acoustic transmission via space-time modulated membranes

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ABSTRACT

Non-reciprocity has recently attracted considerable attention as it enables new possibilities in wave manipulation and control. Here, we propose and analyze theoretically and numerically a waveguide system consisting of two membranes whose surface tensions are time-modulated with a phase difference between them. Strong non-reciprocity and low insertion loss can result for waves transmitted through the structure. An analytic approach is developed to calculate the harmonics generation in the system. Based on this approach, the optimal design of a two-membrane system for non-reciprocal wave behavior is then discussed. By suitably choosing the modulation parameters, the isolation factor for waves incident from opposite sides can reach as high as 19.8 dB and an insertion loss of only 2.8 dB, with an overall dimension being less than 1/3 wavelength. These theoretical results are verified by time-dependent finite element simulations. Our work provides a feasible way to design acoustic non-reciprocal devices.

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In wave dynamics, the transmission from one port to another generally remains the same when the two ports are switched, based on the reciprocity theorems.[1–3] Breaking reciprocity in wave propagation, on the other hand, offers interesting possibilities for a number of applications and thus attracts considerable attention in a variety of fields. In electromagnetics, non-reciprocity is an important scientific and technological concept and has produced several of useful devices.[4–10] Optical isolators,[7] for example, are important components which can be used in protecting a laser from reflections or mitigating multipath interference in an optical communication system. In acoustics, the realization of non-reciprocal acoustic transmission has also become an active research area,[11] and many non-reciprocal acoustic devices have been proposed, such as acoustic diodes/rectifiers,[12–16] insulators,[17–20] circulators,[21] and parametric converters and amplifiers.[22] Additionally, non-reciprocity is also a very active area in elastic wave manipulation[23–25] and thermal rectification.[26,27] These devices can have applications in areas including imaging, communications, and energy harvesting.

In acoustics, one approach to realize non-reciprocal transmission is introducing nonlinearity into the acoustic system.[12–15] However, these non-reciprocal acoustic devices based on nonlinearity have some intrinsic disadvantages. For example, to realize strong non-reciprocity, a high level of input energy is typically needed to induce enough nonlinear effects. Moreover, signals would be severely distorted at the output, and the frequency content is changed. For certain applications, linear non-reciprocity is preferred, which hinders the practical usage of this nonlinear approach.

Fundamentally linear non-reciprocal acoustic devices have also been explored. One kind of these devices is designed by adding an external flow field into the acoustic system. Acoustic circulators[21] and topological insulators[17,18] have been realized based on this approach. However, these circulating fluid based devices typically require high-level energy input and sophisticated control. They also suffer from a high level of noise generated by the fans and flowing fluid.

Recently, space-time modulation has emerged as an alternative means to achieve non-reciprocal acoustic transmission.[16,19,20,22,28] The use of space-time modulation can create a directional bias that breaks time-reversal symmetry and therefore leads to non-reciprocal transmission. As only a few time-modulated elements are required to induce a time-odd bias, this kind of devices can be very compact, while it still exhibits strong non-reciprocity. Furthermore, space-time modulation systems offer many degrees of freedom and can provide functionalities that go beyond non-reciprocal transmission by correctly designing the modulation strategy.

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In this paper, we propose and analyze a waveguide system consisting of membranes whose surface tensions are time-modulated. An analytical approach is developed to calculate the harmonics generation in the system. By further introducing spatial directional bias into the system, non-reciprocal acoustic transmission is realized. A search for the optimal, maximally nonreciprocal design of the two-membrane system based on our analytical approach is performed. It is found that with this two-membrane system, the isolation factor for incident waves transmitting from opposite sides can reach as high as 19.8 dB with a 2.8 dB low insertion loss. The overall dimension of the structure is less than 1/3 wavelength. Time-dependent finite element simulations are provided to verify the theoretical calculation. Such compact non-reciprocal acoustic devices are of great interest in areas across communication and sensing.

We begin with analyzing a single membrane under time-modulation as shown in Fig. 1(a). The radius of the waveguide is R. The background medium is air with density ρ_0 and sound speed c_0. The membrane is made of silicone rubber with density ρ_m, Young modulus E_m, and shear modulus G_m. The radius and thickness of the membrane are R and d, respectively. The membrane is fixed around the edge with time-varying surface tension $T = T_0[1 + m\cos(\Omega t - \phi)]^2$. The surface tension function here needs to be squared because sinusoidally modulated sound speed and impedance are needed. In practice, the required surface tension modulation can be realized by applying AC voltage on piezoelectric membranes, which shows promising aspects in terms of feasibility. Here, m is the modulation depth, Ω is the modulation angular frequency, and ϕ is the initial phase of the modulation. One-dimensional wave propagation along the x axis is considered, and the membrane is located at $x = 0$. The pressure fields of the incident, reflected, and transmitted waves are denoted as p_i, p_r, and p_t, respectively. It is assumed that the membrane with modulated tension will not emit acoustic waves. For an incident wave with angular frequency ω and pressure p_0, the acoustic waves in the waveguide can be expressed as the sum of harmonics

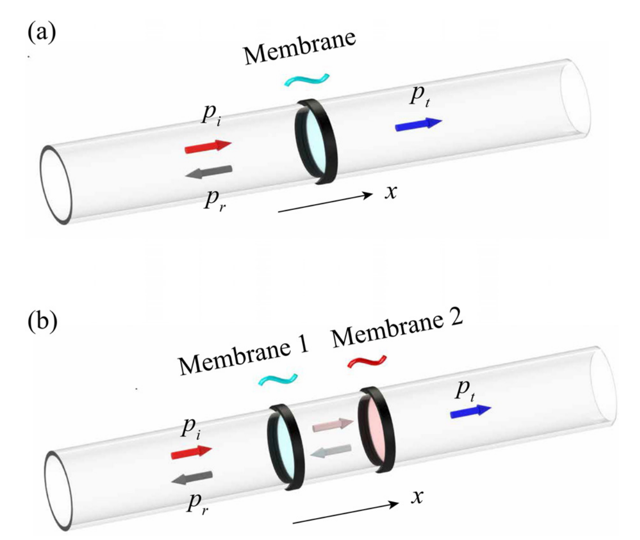

FIG. 1. Schematic of (a) a single-membrane system and (b) a two-membrane system with modulation. p_i: incident wave, p_r: reflected wave, and p_t: transmitted wave.

$$\begin{cases} p_i(x) = p_0 \exp[j(\omega t - kx)], \\ p_r(x) = \sum_n r_n p_0 \exp[j(\omega_n t + k_n x)], \\ p_t(x) = \sum_n t_n p_0 \exp[j(\omega_n t - k_n x)], \end{cases} \tag{1}$$

where r_n and t_n are the reflection and transmission coefficients of the nth mode, respectively. In our time-modulated system, the harmonics ω_n are $\omega_n = \omega + n\Omega$. The wave vector of each mode can therefore be expressed as $k_n = \omega_n / c_0$. By applying the relation $\rho_0 \frac{\partial v}{\partial t} = -\frac{\partial p}{\partial x}$, the acoustic velocity fields can be obtained from the corresponding pressure fields

$$\begin{cases} v_i(x) = \frac{p_0}{z_0} \exp[j(\omega t - kx)], \\ v_r(x) = \sum_n -\frac{r_n p_0}{z_0} \exp[j(\omega_n t + k_n x)], \\ v_t(x) = \sum_n \frac{t_n p_0}{z_0} \exp[j(\omega_n t - k_n x)], \end{cases} \tag{2}$$

where $z_0 = \rho_0 c_0$. At $x = 0$, velocity continuity yields

$$\delta_n - r_n = t_n, \tag{3}$$

where δ_n is the Kronecker delta. On the other hand, the acoustic pressure and velocity at $x = 0$ should satisfy the impedance relation

$$Z_m(v_i + v_r) = Z_m v_t = (p_i + p_r) - p_t, \tag{4}$$

where $Z_m = -j\omega\rho_m d[J_0(\frac{\omega R}{c_m})]/[J_2(\frac{\omega R}{c_m})]$ is the impedance of the membrane.[29] J_0 and J_2 are the 0th order and 2nd order Bessel functions of the first kind. We can expand the impedance at c_{m0} as

$$Z_m = Z_{m0} + \frac{mZ_{v0}}{2}\{\exp[j(\Omega t - \phi)] + \exp[-j(\Omega t - \phi)]\}, \tag{5}$$

where $Z_{m0} = Z_m|_{c_m = c_{m0}}$, $Z_{v0} = c_{m0}\frac{\partial Z_m}{\partial c_m}|_{c_m = c_{m0}}$, and $c_{m0} = \sqrt{\frac{T_0}{\rho_m d}}$. Inserting Eqs. (2), (3), and (5) into Eq. (4), we can get the following equation:

$$m\frac{Z_{v0}(\omega_{n-1})}{2z_0}\exp(-j\phi)t_{n-1} + \left[\frac{Z_{m0}(\omega_n)}{z_0} + 2\right]t_n + m\frac{Z_{v0}(\omega_{n+1})}{2z_0}\exp(j\phi)t_{n+1} = 2\delta_n. \tag{6}$$

The parameters in our single membrane system are given as $\rho_0 = 1.21$ kg/m^3, $c_0 = 343$ m/s, $\rho_m = 1300$ kg/m^3, $E_m = 117.5$ kPa, $G_m = 40$ kPa, $R = 7.5$ mm, $d = 0.2$ mm, $T_0 = 3$ MPa $\times$ d, $m = 0.1$, $f_m = 200$ Hz, and $\Omega = 2\pi f_m$, $\phi = 0$. The modulation frequency is chosen for two factors as follows: First, a small modulation frequency is generally preferred as slow modulation is typically easier to realize. Second, a modulation frequency away from the frequency of interest can help avoid potential frequency mixing of the modulation signal and make it easier for the analysis. For weak modulation $m = 0.1$, the conversion from the fundamental mode to higher order harmonics ($|n| \geq 3$) can be omitted, and only $n = 0, \pm 1$, and ± 2 are considered in Eq. (6). Solving Eq. (6), we can get the transmission spectrum

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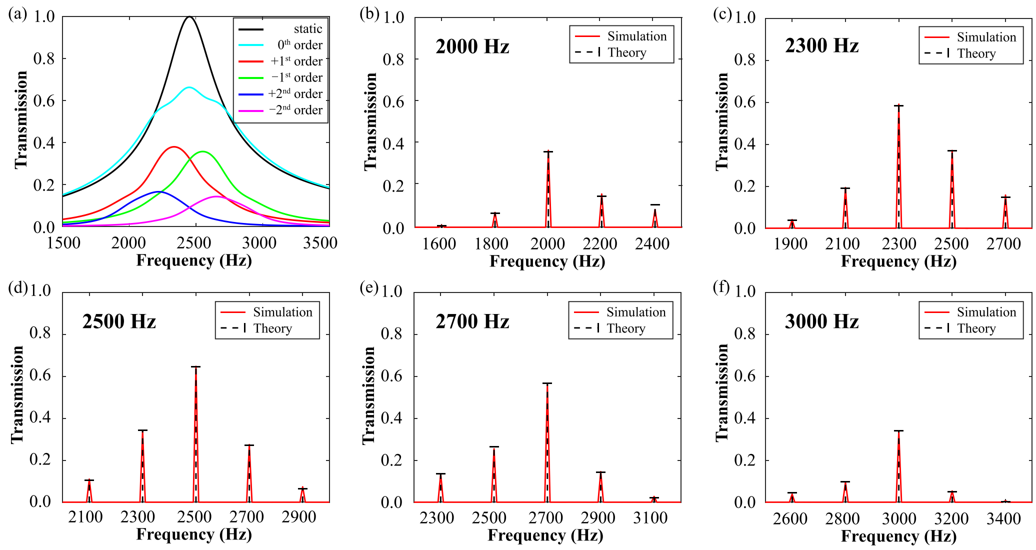

FIG. 2. Transmission through a single temporal modulated membrane. (a) Analytically calculated transmission in the static case and under the time modulation of different modes. (b)–(f) Comparison of transmission of different modes in theory and simulation at different frequencies. Red solid curve: transmission spectrum obtained from time-dependent finite element simulation. Black dashed curve: theoretically predicted transmission at the 0th, ±1st, and ±2nd orders. Higher order harmonics are truncated in derivation.

directly as shown in Fig. 2(a). It can be seen that under the temporal modulation, a part of the 0th mode is converted to the ±1st and ±2nd orders around the resonant frequency of the membrane. The conversion efficiency is higher near the resonant frequency. This indicates that non-reciprocal transmission with low insertion loss may be possible as the amplitude of the 0th mode is largely preserved. Numerical simulations using COMSOL 5.3 are performed to verify the analytic results. A series of simulations are carried out with different excitation frequencies, and the response of the time-modulated membrane is analyzed using Fourier transform. The results are summarized in Figs. 2(b)–2(f), and good agreement between theory and simulation can be observed. It not only verifies the theoretical model but also validates the approximation of truncating the higher order harmonics ($|n| \geq 3$).

Then, we introduce spatial bias into the time-modulated membranes system, as shown in Fig. 1(b). Two membranes are located at $x=0$ and $x=D$. $D=20$ mm, and their tensions are modulated as $T_1 = T_0[1+m\cos(\Omega t-\phi_1)]^2$ and $T_2 = T_0[1+m\cos(\Omega t-\phi_2)]^2$, respectively. All the other parameters are the same as those in the single membrane system. Following the same procedures in the single membrane system, we can get the transmission of the system by applying velocity continuity and impedance relation at $x=0$ and $x=D$, respectively,

$$\begin{aligned}&\left\{m\frac{Z_{v0}(\omega_{n-1})}{2z_0}\exp(-j\phi_1)a_{n-1}-m\frac{Z_{v0}(\omega_{n-1})}{2z_0}\exp(-j\phi_1)b_{n-1}\right\}\\&+\left\{\left[\frac{Z_{m0}(\omega_n)}{z_0}+2\right]a_n-\frac{Z_{m0}(\omega_n)}{z_0}b_n-2\delta_n\right\}\\&+\left\{m\frac{Z_{v0}(\omega_{n+1})}{2z_0}\exp(j\phi_1)a_{n+1}-m\frac{Z_{v0}(\omega_{n+1})}{2z_0}\exp(j\phi_1)b_{n+1}\right\}=0,\end{aligned} \tag{7}$$

$$\begin{aligned}&\left\{m\frac{Z_{v0}(\omega_{n-1})}{2z_0}\exp(-j\phi_2)a_{n-1}\exp(-jk_{n-1}D)\right.\\&\left.-m\frac{Z_{v0}(\omega_{n-1})}{2z_0}\exp(-j\phi_2)b_{n-1}\exp(jk_{n-1}D)\right\}\\&+\left\{\frac{Z_{m0}(\omega_n)}{z_0}a_n\exp(-jk_nD)-\left[\frac{Z_{m0}(\omega_n)}{z_0}+2\right]b_n\exp(jk_{n-1}D)\right\}\\&+\left\{m\frac{Z_{v0}(\omega_{n+1})}{2z_0}\exp(j\phi_2)a_{n+1}\exp(-jk_{n-1}D)\right.\\&\left.-m\frac{Z_{v0}(\omega_{n+1})}{2z_0}\exp(j\phi_2)b_{n+1}\exp(jk_{n-1}D)\right\}=0,\end{aligned} \tag{8}$$

where a_n and b_n are the pressure coefficients of forward and backward waves associated with the nth mode in between two membranes. Finally, the transmission of the system can be obtained by using velocity continuity at $x=d$, which yields

$$t_n = a_n\exp(-jk_nD) - b_n\exp(jk_nD). \tag{9}$$

In this two-membrane system, the time-odd bias is realized by imposing a phase difference $\Delta\phi = \phi_1 - \phi_2$ between the modulated surface tensions of two membranes. It will be shown that temporal modulation together with the phase difference will lead to non-reciprocal transmission for opposite directions. Here, we are interested in the transmission coefficients t_0^+ and t_0^- of the fundamental mode, where t_0^+ and t_0^- are the transmission coefficients of the positive and negative directions, respectively. To evaluate the performance of the system, two metrics are introduced: isolation factor IF and insertion loss IL. Here, $IF = 20\log|t_0^+/t_0^-|$ and $IL = -20\log|t_0^+|$ measure the degree of non-reciprocity and attenuation in the positive direction, respectively. Figure 3 depicts the variation of these two parameters as a function of incident frequency and phase difference. It can be seen that

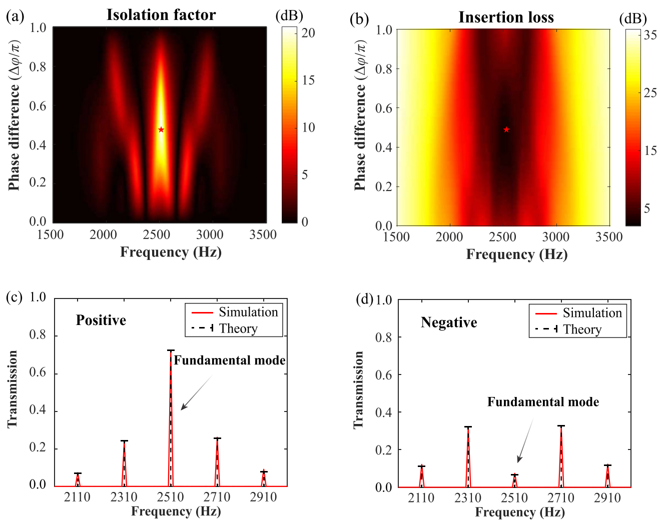

FIG. 3. (a) Isolation factor and (b) insertion loss of the two-membrane system under space-time modulation with modulation frequency $f_m = 200$ Hz and modulation depth $m = 0.1$. The red star marks the parameters implemented in the following theoretical and simulated analysis. Comparison of transmission of different modes in theory and simulation in (c) the positive direction and (d) the negative direction. Red solid curve: transmission spectrum obtained from time-dependent finite element simulation. Black dashed curve: theoretically predicted transmission at the 0th, ±1st, and ±2nd orders. Higher order harmonics are truncated in derivation.

a band of strong non-reciprocity is opened near the resonant frequency with a moderate value of phase difference. Besides, the maximum of *IF* and minimum of *IS* appear in the same place, which means that we can achieve strong non-reciprocity and low attenuation simultaneously.

Then, we choose the optimal incident frequency $f_0 = 2510$ Hz and phase difference $\Delta\phi = 0.512\pi$ as marked by red stars in Figs. 3(a) and 3(b). The modulation frequency is 200 Hz, and the modulation depth is 0.1. The theoretically calculated transmission coefficients of the generated harmonics for positive and negative directions are depicted by the black dashed line in Figs. 3(c) and 3(d), respectively. A clear non-reciprocal transmission is observed. It can be seen that the 0th (2510 Hz) transmission coefficients for opposite directions are 0.728 and 0.070, respectively, which correspond to an isolation factor 20.3 dB and an insertion loss 2.8 dB. To further demonstrate non-reciprocal transmission, numerical simulations are performed using the designed parameters. A sinusoidal wave is incident into the waveguide from one end, and a modulated signal is obtained from the other end. Applying Fourier transform, we can get the signal spectrum as depicted by the red solid line in Figs. 3(c) and 3(d). In the positive direction, the transmission coefficient amplitude of the fundamental mode (2510 Hz) is 0.725, while in the negative direction, the transmission coefficient amplitude is only 0.074. The corresponding isolation factor is 19.8 dB, and the insertion loss is 2.8 dB. The results from theoretical calculation and simulation are in excellent agreement, which again validates the theoretical model.

To conclude, a time-modulated membrane system is proposed to achieve non-reciprocal acoustic transmission. An analytic method based on mode expansion is developed to calculate the frequency conversion in a system with time modulation only. The theoretical calculation results agree well with the time-dependent finite element simulation. Non-reciprocal acoustic transmission is achieved in the two-membrane system by introducing a phase difference in the time modulation of two membranes. By carefully choosing the input frequency and modulation phase difference, the isolation factor for incident waves transmitting from two sides of the system can reach as high as 19.8 dB with an insertion loss of only 2.8 dB, which is much better than the two-Helmholtz resonator system.[20] This strong non-reciprocity with low insertion loss is attributed to the sound transmission characteristics of the membrane. That is, in the time-modulated membrane system, the influence of time modulation is stronger near the resonant frequency while the high transmission of the fundamental mode is maintained. Second, as the resonant frequency of the membrane depends on the surface tension, the operation frequency can be conveniently tuned by modifying the surface tension without changing the physical configuration of the system, which broadens the practical usage of the device. Moreover, the surface tension modulation would not radiate acoustic waves as the movement-based modulation always does,[20,22] which can avoid the high-level noise from the modulation-driven motion of the element. This nonradiative characteristic is of great importance in the experimental demonstration of the device and its application in practice. In this work, we only study single-membrane and two-membrane systems. By using more than two membranes, non-reciprocity can, in principle, be much stronger, and other interesting phenomena may appear. For example, a series of membranes may be used for effective density modulation to achieve mode conversion and parametric amplification.[22] Our work demonstrates a feasible implementation for the design of non-reciprocal acoustic devices with better performance and simpler configurations, which offers a design platform for a number of applications, including imaging, communications, and energy harvesting.

X.Z. and J.L. contributed equally to this work.

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